

HIGHER ORDER THINKING THROUGH CALCULUS FOR KIDS

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The Calculus for Kids project has run over four years in five Australian states with 227 students in 18 schools.

Participating students were 10-12 years old and studied integral calculus using computer algebra software (MAPLE). Their success in a post-test shows levels of achievement comparable to first year university engineering students. The project demonstrates how purposeful computer use can engender higher order thinking and provides exemplary evidence for systematic curriculum re-design in an era of ubiquitous information technology. The results in this report showed the learning outcomes were independent of student gender but responses to application questions were related to school rurality (based on ICSEA value). This makes the approach more attractive for general adoption and strengthens the argument for considering parallel developments in other topic areas.

Introduction

Roschelle et al. (2000) highlighted how supportive structural conditions within schools are essential for the effective use of computers to enhance student learning. As the authors suggest, the introduction of computers in schools alone does not improve student performance. Instead, computers must be seen as a learning tool to be used in combination with effective curriculum, pedagogy and assessment. These findings are consistent with those of meta-analyses conducted by Hattie (2008), and Tamim et al. (2011), both of which suggest computers are most effective when used in support of appropriate curriculum goals and teaching strategies. To provide the conditions necessary to support the effective use of computers, Roschelle et al. (2000) argue, schools must

increase professional development opportunities for teachers, modernise their curriculum, and change their assessment practices from testing rote-learned concepts to tasks which require students to provide solutions to complex problems using higher-order thinking skills.

Since Kelman (1989) proposed the use of computers to support higher-order thinking, concerted research attention has investigated the effectiveness of computer technology for engaging students in curriculum-based tasks which promote higher order thinking. Roschelle et al. (2000), for example, examined the effectiveness of the SimCalc program, a technology-enhanced mathematics program, for improving students' higher-order thinking and learning of algebra. The SimCalc program involved the use of interactive computer software in middle school mathematics classes. Using the software, students could engage with and manipulate graphs and reflect on the resultant changes to the algebraic equations they represented. In addition to providing the software, the research team supplied curriculum materials, such as lesson plans, student workbooks and relevant computer files, and facilitated a range of professional development opportunities for teachers. The use of the computer software, therefore, was supported by relevant mathematics curriculum and pedagogy. Across three large-scale studies of the learning gains experienced by Year 7 (1st year $n = 1,444$, 2nd year $n = 997$) and Year 8 students ($n = 657$), the researchers found the SimCalc program significantly increased students' learning in algebra (effect sizes of 0.63, 0.50 and 0.56, respectively) (SRI International, 2011).

While these results provide positive support for the use of SimCalc for improving students' algebra learning, it is important to note that a range of other variables were also shown to

$$x(1+e^{-x}) = e^{-x} \frac{x}{1-x} = e^2$$

have an effect on student learning, albeit an inconsistent effect between the studies. In the second study, for example, the use of SimCalc was a negative predictor of student learning gains in schools with socio-economically disadvantaged students. Ethnicity was also found to have a relationship to student learning gains in the second study with Hispanic students demonstrating smaller learning gains than Caucasian students. Furthermore, students with greater prior knowledge of algebra tended to complete more of the exercises in their workbooks and with greater accuracy than peers with less prior knowledge. The researchers subsequently found workbook completion (using the software) and accuracy to be an important predictor of student learning gains (SRI International, 2011). These findings continue to highlight how the effectiveness of computer technologies for learning can be mediated by range of other, often structural, factors.

More recent research examining the use of SimCalc has further demonstrated the effectiveness of the program for improving students' algebra learning. Hegedus, Dalton & Tapper (2015) evaluated the use of SimCalc in high school classes (15-17 year old students). 606 students participated in the first year evaluation of SimCalc while a further 293 students participated in the second year replication study. In each year of the study, the researchers found students who used SimCalc recorded higher relative gains in post-test scores than students who did not use the software. In Year 1, SimCalc students recorded relative gains of 25% (standard deviation = 0.49) compared to students in the control group (11%, standard deviation = 0.47). The replication study (Year 2) recorded similar gains: 26% gains for SimCalc students (standard deviation = 0.50) and 14% gains for control students (standard deviation = 0.47). These findings support the conclusion that SimCalc is effective in increasing students' learning.

This paper further describes the *Calculus for Kids* project (Chin, Fluck, Ranmuthugala & Penesis, 2011; Fluck, Ranmuthugala, Chin & Penesis, 2012) where computer software is used to teach primary school students integral calculus, normally taught in Year 12. This research provides us with information on how these students cope with real-world application problems and their ability to apply to other problems. The Calculus for Kids project differed from research using SimCalc by expecting students to use computers during the post-test.

Literature

The notion of a 'technological gender gap' between male and female computer-related behaviours and performance,

attitudes towards computers and self-efficacy for completing ICT tasks has received considerable research attention over the past twenty years (Broos, 2005; Colley & Comber, 2003; Zhong, 2011). Whitley's (1997) meta-analysis reviewed 82 American and Canadian studies which claimed to have found gender differences in computer-related behaviours and attitudes in adult, college, high school, middle school and elementary school student populations. Although effect sizes varied within these individual studies, Whitley's analysis determined the largest effect sizes were generally found for high school students and attributed this, in part, to a "socialisation process" which historically portrayed computer use as the domain of boys and men. Whitley's analysis also revealed, although gender differences were found to be statistically significant, the effect size was small ($d = 0.326$), suggesting the result may even be representative of sampling errors or self-selection biases within the reviewed studies.

More recent research has examined gender differences in computer behaviours, attitudes and self-efficacy using national and international student performance data. Tømte & Hatlevik (2011) utilised data from the 2006 Programme for International Student Assessment (PISA) Information and Communication Technology survey to describe the relationship between computer self-efficacy and gender in 15-year-old Norwegian and Finnish students ($n = 9,400$). The survey asked students to self-report on their ability to complete a series of 14 different ICT tasks. While the authors did find some evidence of gender differences related to ICT self-efficacy it is significant to note both male and female Norwegian students reported higher levels of ICT self-efficacy than all Finnish students. At the national level, Finnish males reported higher levels of ICT self-efficacy than Finnish females, while Norwegian females reported higher levels than Norwegian males. The conflicting nature of these results and the localised context from which the data for this study was drawn means further research is warranted to ensure the findings can be generalised to a wider population.

In Australia, findings derived from the 2014 National Assessment Program (NAP) ICT literacy assessment provide evidence of gender differences in ICT literacy test scores. The assessment tested the ICT literacy of 11,000 Australian students in Year 6 and Year 10. In the test, girls out-performed boys in both year levels (on average girls' scores were 23 points higher than boys in Year 6 and 29 points higher in Year 10) (Australian Curriculum, Assessment and Reporting Authority, 2015). These results stand in contrast to older research claiming boys are more skilled in ICT (Attewell & Battle, 1999; Imhof, Vollmeyer, & Beierlein, 2007; Zhong, 2011). The NAP results are more consistent with very recent

findings from Belgium (Aesaert & Van Braak, 2015) and the United States (Hohlfeld, Ritzhaupt, & Barron, 2013) which also found girls out-performed boys on ICT skills tests.

Our enquiry was to see if using computers made it possible for such young students to understand integral calculus and apply their understanding to solve mathematical and real world problems. In addition, we were interested in the ways the genders approached learning in this computer-intensive environment, and whether they differed in their achievements.

Methodology

The project primarily focuses on teaching integral calculus to Year 6 primary school students (aged 10 to 12 years) recruited in Australian schools in Tasmania, Victoria, South Australia, New South Wales and Queensland. Research ethics for this study was first sought and approved by the individual states. An expression of interest was then sent to schools to recruit the teachers. Once the schools confirmed their commitment, a teacher from each of these schools were flown to Launceston and trained in a one-day workshop. The computer software used is MAPLE® as in the first and second year Mathematics units in the Bachelor of Engineering (Maritime) programs at the University of Tasmania. This software was chosen for its simplicity, WYSIWYG and user-friendly configuration. Most importantly, it retains classical mathematical notation, unlike most other computer algebra software. During the workshop, the teachers are trained to deliver a total of 13 lessons. For each lesson, the teachers are provided with an interactive PowerPoint presentation, a Maple worksheet datafile and a PDF worksheet document containing basic questions and real world application problems. Teachers are also provided with an answer booklet illustrating the worksheet responses students should obtain for all 13 lessons. The teachers return to their schools to implement the lessons with students.

Unlike many other educational investigations, a pre-test of ability was not conducted. There were two reasons for this. Firstly, this is a fairly short intervention study, and one of the lessons was expected to be used for a post-test of learning achievement and attitudes to learning mathematics. That amounts to nearly 8% of the learning time. Doubling the test time to 15% (or adding another lesson) would need to be well justified. The second reason for omitting a pre-test, was the topic. It was generally agreed that none of the students could be expected to have been taught or to know anything about integral calculus. The very small chance that one or two would register a score on a pre-test of the topic did not seem sufficiently likely to justify the demoralising effect on all the other students. Therefore the pre-test was designed out of the

learning sequence because it had little value and could demoralise students.

The students are first taught the basics of how to "drive" the software in order to perform simple calculations. Then an introduction is given on plotting points, lines and defining a function. In order for students to appreciate integral calculus and to address past comments from reviewers, two lessons were added to the program. These lessons focus on the formation of equations found within the program. To help students understand where the equations come from, we start with a basic parabola $y = x^2$ and then extend this to the general equation of a parabola $y = a(x - h)^2 + b$. This is accompanied with graphical explanations. See Figure 1 for an example.

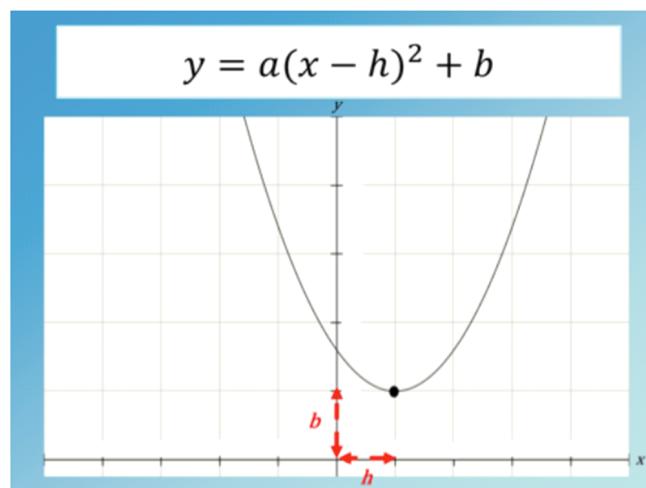


Figure 1: Graphical representation for the general equation of a parabola

The following lesson required students to formulate the equation of the parabola based on the given parameters. An application problem is given in Figure 2 where students are required to determine the equation that models the cables of the Golden Gate Bridge. This part of the program equips students to derive (in some limited circumstances) the function equation that describes a particular curve. Thus it more closely parallels conventional courses in integral calculus, and provides a better demonstration of higher order thinking which corresponds to 'creating/origination' in Bloom's taxonomy (1956). In solving this problem, students have to super-impose a Cartesian grid onto the diagram, then work out the coordinates for the turning and one other point. They substitute these values into the general equation for a parabola, leaving one unknown variable to be found. This is calculated by using the 'SOLVE' function in MAPLE, and so the precise equation for this practical example can be found.

$$1 + e^2) = e^2$$

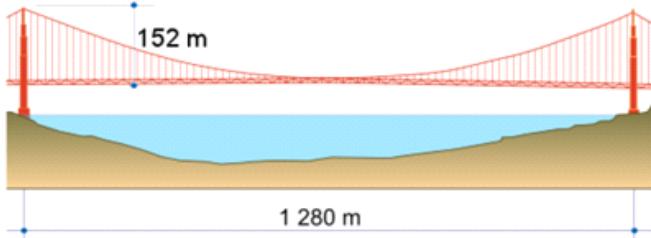


Figure 2: Example of an application problem

Once these preliminaries are delivered, students are introduced to the concept of integration and shown where integral calculus can be applied. An integral application question is shown in Figure 3 where students are asked to determine the equation of the parabola and the amount of fabric needed for the end section to make the tent. In the preceding weeks leading up to the topic on integral application questions, students have already learned that to find the area under a curve, they need to perform integration. They were taught what the limits (lower and upper limits) of integration are. Combining this knowledge and the previous knowledge learnt in setting up the equation of a parabola, they are then able to find the area underneath the curve and hence determine the amount of fabric required.

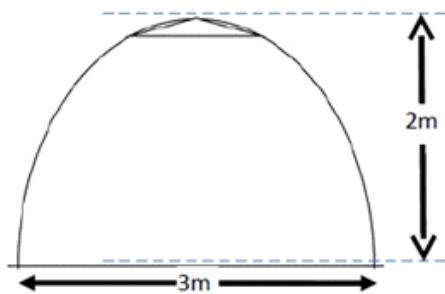


Figure 3: Example of an integral application problem

Further sample application questions can be found in articles published by the co-authors (Fluck, Chin, Ranmuthugala & Penesis, 2014; Chin, Fluck, Ranmuthugala & Penesis, 2011; Fluck, Ranmuthugala, Chin & Penesis, 2012; Penesis, Chin, Ranmuthugala & Fluck, 2011) amongst others.

Members of the research team visited each school about half way through the program to observe the class, ensure the program was on track, assist with any computer difficulties and obtain feedback from the teacher and students. These visits are vital for the continual refinement of the resources where needed. The research team also monitored progress via telephone conferences and emails with the teachers to ensure smooth progression of the program. Figure 4 shows a sign

located on the oval of the school. This attracted attention and interests from parents, the public and other neighbouring schools.

In the final stage of the program, all students are required to take a test to assess their knowledge and skills that they have gained throughout the programme. The test is based on questions drawn from first year maritime engineering mathematics examination papers. The test consists of 15 assessed questions, of which there are 7 application questions. Immediately after the test, an attitudinal test was also administered. This included a Mathematics self-efficacy scale (Tapia & Marsh, 2004) and a Mathematics and Technology Attitudes Scale (Barkatsas, 2004; Pierce, Stacey & Barkatsas, 2007).

The completed MAPLE files and surveys were collected and sent back to the research team for analysis. By return, schools and students receive a short descriptive certificate showing their learning achievement and attitude to learning mathematics relative to the means for the class.



Figure 4: School sign

Results and Discussions

One of the main aims of this research is to determine whether the students are able to transfer higher level concepts and skills and apply them to real world application problems using appropriate technology. To date, a total of 227 students in 18 schools have participated in the Calculus for Kids project. This project spanned five Australian states which included Tasmania (pilot project), Victoria, New South Wales, Queensland and South Australia.

$$\frac{1}{1-x} = e^x$$

Table 1: Student demographics

State	Location	ICSEA	Total Scores			Scores – Females			Scores – Males			Application Questions		
			n	M	σ	n	μ	σ	n	μ	σ	n	μ	σ
NSW	Rural	911	18	70.3	16.6	9	75.9	15.4	9	64.7	16.7	18	59.72	18.81
QLD	Urban	1027	90	66.1	17.1	42	66.9	16.0	48	65.4	18.1	90	51.31	20.67
SA	Rural	945	21	51.0	19.2	6	43.9	18.6	15	53.9	19.3	21	28.23	22.64
	Urban	1001	22	63.4	19.1	9	69.9	8.8	13	58.9	23.1	22	49.03	15.52
VIC	Urban	1092	76	75.9	13.3	35	78.1	10.6	41	74.1	15.2	76	62.59	17.41

Table 1 provides a summary of the student demographics, location, Index of Community Socio-Educational Advantage (ICSEA), mean test scores based on gender and performance in application questions for the schools (excluding Tasmania) involved in the project. Note that the mean ICSEA value is 1000 with a standard deviation of 100 and that values below the mean indicate schools with fewer advantages. Here, *n* is the sample size, μ is the mean and σ is the standard deviation. All data was pooled and analysed using SPSS Version 21 (IBM Corp., 2012). The mean total scores **M** for the whole test, as shown in column 5 of Table 1, are all above the university pass grade of 50%.

Table 2 shows the result of an independent-sample t-test which was conducted to compare the test scores for students from rural and urban areas. The results showed that there was significant difference in scores for rural students ($M=42.77$, $SD=26.10$) and urban students ($M=55.60$, $SD=19.64$; $t(47)=2.91$, $p=0.006$, two tailed). The results show that the schools' location has a significant difference when comparing the mean scores, because the t value is above the critical value. This could be due to urban schools are better staffed, have better facilities and study environment, and students subjected to positive study habits.

Table 2: Result of t-test analysis of the influence of school location on performance of application questions

Location	n	μ	σ	t-value	degrees of freedom
Rural	39	42.77	26.10		
Urban	188	55.60	19.64	2.91*	47

*Significant at 0.05 level

On whether gender plays a part in the performance, an independent-sample t-test was conducted to compare the test scores for male and female students. As shown in Table 3, there was no significant difference in scores for male students ($M=51.72$, $SD=21.86$) and female students ($M=55.49$, $SD=20.69$; $t(225)=1.32$, $p=0.19$, two-tailed). This result corroborates research done by Hyde and Mertz (2009) and demonstrates female students are as accomplished as males,

possibly due to the various multi-media techniques used in the presentation and delivery of the teaching materials.

Table 3: Result of t-test analysis of the influence of gender on performance of application questions

Gender	n	μ	σ	t-value	degrees of freedom
Females	101	55.49	20.69		
Males	126	51.72	21.86	1.32*	225

*Significant at 0.05 level

The last question on the test asked students "What is Calculus good for?". Some of the comments gave us a better idea of what students think calculus can be applied to:

integral calculus is useful because we can find area under a curved line and can also find the area of bridges, skate ramps etc by using a method $y=a^(x-h)^2+b$.*

because you can work out the area of the thing you might need to carpet or paint and more everyday things.

because it gives us a clue on time velocity and speed which is related to real life.

For jobs such as engineering, construction or building any object of any kind.

Because if you want to become an engineer you will not do simple shapes like triangles and squares you will be doing complicated shapes and those shapes are in calculus. So if we learn calculus young then we will have less difficulty when we are older and we are doing calculus. So calculus helps us learn the real world and shapes that we will actually use if we become engineers or architects so that's why it is useful.

It was quite apparent these primary school students had acquired a substantial understanding of the mathematical techniques of integral calculus, and more importantly, their potential applications. They were ready to make the leap into learning at a higher level. But are schools, teachers and curriculum designers ready to incorporate computers into learning programs to this degree?

Conclusion

The results from this Calculus for Kids project has shown us that using computers can transform how students learn mathematics. The results indicate students are able to understand the concepts and apply them to the relevant application problems from analogical reasoning rather than plain memorisation. This is supported by research done by Richland and Simms (2015). Of particular importance is that by

$$(1-x)y = \sin y$$

using the MAPLE software and multi-media teaching materials to assist visualisation, students were able to master concepts at a far younger age than considered appropriate for this subject matter. It is tempting to speculate what other academic areas could be similarly developed to allow a leap forward with curricula. As cited in the introduction, computers can make a

big difference in school education if they are used in the context of "appropriate curriculum goals and teaching strategies". Following current trends for ubiquitous computer use in classrooms, gender-neutral demonstrations of higher order thinking such as Calculus for Kids are an important beacon into the future.

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